Similarity Solution for Unsteady MHD Flow near a Stagnation Point of a Three-Dimensional Porous Body with Heat and Mass Transfer, Using HPM

Vivek Kumar Sharma[1], Aisha Rafi[1]

[1]Department of Mathematics, Jagan Nath University, Jaipur, Rajasthan, India

Abstract: The problem of unsteady mixed convection heat and mass transfer near the stagnation point of a three-dimensional porous body in the presence of magnetic field, chemical reaction and heat source or sink is analyzed. Similarity transformation and Homotopy Perturbation Method are used to solve the transformed similarity equations in the boundary layer. Velocity distribution and temperature distribution are shown through graphs for various physical parameters and coefficients of skin friction and heat transfer are present through tables.

Keywords: similarity transformation, Homotopy Perturbation Method, stagnation point; heat source/sink.

Introduction: Hydro magnetic incompressible viscous flow has many important engineering applications such as magneto hydrodynamic power generators and the cooling of reactors. The laminar flow above a line heat source in a transverse magnetic field was studied by Gray (1997).Vajravelu and Hadjinicolaou (1997) made analysis to flows and heat transfer characteristics in an electrically- conducting fluid near an isothermal sheet. Chamkha (2003) studied the problem of MHD flow of a uniformally stretched vertical permeable surface in the presence of heat generation/absorption and chemical reaction. Cheng and Huang (2004) considered the problem of unsteady flows and heat transfer in the laminar boundary layer on a linearly accelerating surface with suction or blowing in the absence and presence of a heat source or sink. Unsteady heat and mass transfer from a rotating vertical cone with a magnetic field and heat generation or absorption effects was studied by Chamkha and Al-Mudhaf (2005). Chamkha et al. (2006) presented analysis of the effect of heat generation or absorption on thermophoretic free convection boundary layer from a vertical flat plate embedded in a porous medium.. Liao (2006) obtained an accurate series solution of unsteady boundary layer flows over an impulsively stretching plate uniformly valid for all non-dimensional times. Bararnia et al. (2009) investigated analytically the problem of MHD natural convection flow of a heat generation fluid in a porous medium. Sharma and Singh (2009) presented a numerical solution for the problem of effects of variable thermal conductivity and heat source / sink on MHD flow near a stagnation point on a linearly stretching sheet.

The main objective of this paper is to study the effects of heat generation and chemical reaction on unsteady MHD flow heat and mass transfer near a stagnation point of a three dimensional porous body in the presence of a uniform magnetic field. An efficient, similarity transformation and Homotopy Perturbation Method are used to solve the transformed similarity equations in the boundary layer.

The Homotopy Perturbation Method is a combination of the classical perturbation technique and Homotopy technique, which has eliminated the limitations of the traditional perturbation methods. This technique can have full advantage of the traditional perturbation techniques. He J.H. (1999) "Homotopy Perturbation technique,".He J.H. (2003) "Homotopy Perturbation Method: a new nonlinear analytical technique,".Dehghan M. and Shakeri F. (2008) "Use of He's Homotopy Perturbation Method for Solving a Partial Differential Equation Arising in Modeling of Flow in Porous Media,".To illustrate the basic idea of the Homotopy Perturbation Method for solving nonlinear differential equations, we consider the following nonlinear differential equation:

$$A(u) - f(r) = 0, \qquad (1)$$

Subject to boundary condition

$$B\left(u, \frac{\partial u}{\partial n}\right) = 0, \qquad (2)$$

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Where A is a general differential operator, B is a boundary operator, f(r)is a known analytic function, and Γ is the boundary of the domain Ω.

The operator A can, generally speaking, be divided into two parts: a linear part L and

a nonlinear part N. Equation can be rewritten as follows:

$$L(u) + N(u) - f(r) = 0, \tag{3}$$

By the Homotopy technique, we construct a Homotopy $V(r,p) : \Omega*(0,1) \rightarrow R$ which satisfy

$$H(V,p) = (1-p)[L(v) - L(u_0)] + p[A(v) - f(r)] = 0, \tag{4}$$

$$H(V,p) = L(v) - L(u_0) + p\, L(u_0) + p[N(v) - f(r)] = 0, \tag{5}$$

Where $p \in [0, 1]$ is an embedding parameter and u_0 is an initial approximation of which satisfies the boundary conditions.

$$\left.\begin{array}{l} H(V,0) = L(v) - L(u_0) \\ H(V,1) = A(v) - f(r)] \end{array}\right\} \tag{6}$$

Thus, the changing process of p from zero to unity is just that of v(r, p) from $u_0(r)$ to u(r). In Topology, this is called deformation and $L(v) - L(u_0)$, $A(v) - f(r)]$ are called homotopy. According to the HPM, we can first use the embedding parameter p as a "small parameter," and assume that the solution of can be written as a power series in p:

$$V = V_0 + pV_1 + p^2 V_2 + \ldots\ldots\ldots\ldots \tag{7}$$

Setting p=1 results in the approximate solution of

$$u = \lim_{p \to 1} V = V_0 + V_1 + \ldots\ldots\ldots\ldots \tag{8}$$

The series is convergent for most cases; however, the convergent rate depends upon the

Nonlinear operator A(V). The second derivative of N(V) with respect to V must be small because the parameter may be relatively large; that is, $p \to 1$.

Aim of the paper is to investigate the unsteady mixed convection heat and mass transfer near the stagnation point of a three-dimensional porous body in the presence of magnetic field, Similarity transformation and Homotopy Perturbation Method are used to solve the transformed similarity equations in the boundary layer.

Formulation of the problem: Consider unsteady laminar incompressible boundary layer flow of a viscous electrically conducting fluid at a three-dimensional stagnation point with magnetic field, chemical reaction, heat generation/absorption and suction/injection effects. A uniform transverse magnetic field is assumed to be applied normal to the body surface. The fluid properties are assumed to be constant and the chemical reaction is taking place in the flow. The velocity components of the in-viscid flow over the three dimensional body surface are given by:

$$\frac{\partial u}{\partial x}\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0, \tag{9}$$

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z} = \frac{\partial u_e}{\partial t} + u_e\frac{\partial u_e}{\partial x} + \upsilon\frac{\partial^2 u}{\partial z^2} - \frac{\sigma\beta_0^2(u-u_e)}{\rho} + \frac{x[g\beta(T-T_\infty)+g\beta_c(C-C_\infty)]}{l}, \tag{10}$$

$$\frac{\partial u}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z} = \frac{\partial v_e}{\partial t} + v_e\frac{\partial u_e}{\partial x} + \upsilon\frac{\partial^2 v}{\partial z^2} - \frac{\sigma\beta_0^2(v-v_e)}{\rho} + \frac{cy\,[g\beta(T-T_\infty)+g\beta_c(C-C_\infty)]}{l}, \tag{11}$$

$$\frac{\partial T}{\partial t} + u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} + w\frac{\partial T}{\partial z} = \frac{k}{\rho C_p}\frac{\partial^2 T}{\partial z^2} + \frac{Q_o(T-T_\infty)}{\rho C_p}, \tag{12}$$

$$\frac{\partial C}{\partial t} + u\frac{\partial C}{\partial x} + v\frac{\partial C}{\partial y} + w\frac{\partial C}{\partial z} = D\frac{\partial^2 C}{\partial z^2} - K_c(C - C_\infty), \tag{13}$$

Where C dimensional concentration, C_p specific heat of the fluid, D mass diffusion coefficient, k fluid thermal conductivity, k_c chemical reaction parameter, Q_0 heat generation/absorption coefficient, T temperature, t time, u velocity component in x-direction., u_e free stream velocity component in x-direction., v velocity component in

y-direction, v_e free stream velocity component in y-direction, w velocity component in z-direction, x,y and z longitudinal, transverse and normal directions

Corresponding boundary conditions are

$t = 0: u(x, y, z, t) = u_i(x, y, z), v(x, y, z, t) = v_i(x, y, z), w(x, y, z, t) = w_i(x, y, z),$

$T(x, y, z, t) = T_i(x, y, z), C(x, y, z, t) = T_i(x, y, z),$ (14)

$t > 0: u(x, y, z, t) = 0, v(x, y, z, t) = 0, w(x, y, z, t) = w_w,$ $T(x, y, z, t) = T_w,$

$C(x, y, z, t) = C_w$ at y = 0,   (15)

$t > 0: u(x, y, z, t) = u_e(x, y), v(x, y, z, t) = v_e(x, y), T(x, y, z, t) = T_\infty, C(x, y, z, t) = C_\infty$

at $y \to \infty$ (16)

Assuming,

$$\eta = z \left(\frac{a}{\upsilon(1-\lambda\tau)}\right)^{1/2}, \tau = at, u = ax(1-\lambda\tau)^{-1}f(\eta), v = by(1-\lambda\tau)^{-1}s(\eta), \lambda\tau < 1,$$

$$w = -\sqrt{a\upsilon}\,(1-\lambda\tau)^{-1/2}(f+cs), \theta(\eta) = \frac{(T-T_\infty)}{(T_w-T_\infty)}, \phi(\eta) = \frac{(C-C_\infty)}{(C_w-C_\infty)}, c = \frac{b}{a},$$

$$Re_x = \frac{ax^2}{\upsilon(1-\lambda\tau)}, M = \frac{\sigma\beta_0^2 \upsilon Re_x}{\rho x^2 a^2}, \delta = \frac{Q_0 \upsilon Re_x}{\rho C_p x^2 a^2}, \gamma = \frac{k_c \upsilon Re_x}{x^2 a^2}, \quad f_w = \sqrt{a\upsilon(1-\lambda\tau)}\,w_w.$$ (17)

Using assumptions (17) into equation (10)-(13) yields the following similarity equations

$$f''' + f''\left(f + cs - \frac{\lambda\eta}{2}\right) - f'(\lambda + f') + (\lambda + 1) + M(f' - 1) + \lambda_1\theta + \lambda_2\phi = 0 \quad (18)$$

$$s''' + s''\left(f + cs - \frac{\lambda\eta}{2}\right) - s'(\lambda + cs') + (\lambda + c) + M(s' - 1) + \lambda_1\theta + \lambda_2\phi = 0, \quad (19)$$

$$\theta'' + p_r\theta'\left[f + cs - \frac{\lambda\eta}{2}\right] + \delta\theta p_r = 0, \quad (20)$$

$$\phi'' + \phi'\left(f + cs - \frac{\lambda\eta}{2}\right)s_c - \gamma\phi s_c = 0. \quad (21)$$

Where f dimensionless stream functions, M magnetic field parameter, Re_x Reynolds number, S dimensionless stream function, c ratio of the velocity gradients at the edge of the boundary layer, θ temperature, φ cocentration .

we introduce the following Homotopy,

$$D(f, p) = (1 - p)\left[\left(\frac{d^3f}{d\eta^3} - (\lambda - M)\frac{df}{d\eta}\right) - \left(\frac{d^3f_I}{d\eta^3} - (\lambda - M)\frac{df_I}{d\eta}\right)\right] + p\left[\frac{d^3f}{d\eta^3} + \frac{d^2f}{d\eta^2}\left(f + cs - \frac{\lambda\eta}{2}\right) - \frac{df}{d\eta}\left(\lambda + \frac{df}{d\eta}\right) + \right.$$
$$\left. \lambda + 1 + Mdfd\eta - 1 + \lambda1\theta + \lambda2\phi = 0, \right. \quad (22)$$

$$D(s, p) = (1 - p)\left[\left(\frac{d^3s}{d\eta^3} + (M - \lambda)\frac{ds}{d\eta}\right) - \left(\frac{d^3s_I}{d\eta^3} + (M - \lambda)\frac{ds_I}{d\eta}\right)\right] + p\left[\frac{d^3s}{d\eta^3} + \frac{d^2s}{d\eta^2}\left(f + cs - \frac{\lambda\eta}{2}\right) - \frac{ds}{d\eta}\left(\lambda + c\frac{ds}{d\eta}\right) + \right.$$
$$\left. \lambda + c + Mdsd\eta - 1 + \lambda1\theta + \lambda2\phi = 0, \right. \quad (23)$$

$$D(\theta, p) = (1 - p)\left[\left(\frac{d^2\theta}{d\eta^2} + \delta p_r \theta\right) - \left(\frac{d^2\theta_I}{d\eta^2} + \delta p_r \theta_I\right)\right] + p\left[\frac{d^2\theta}{d\eta^2} + p_r \frac{d\theta}{d\eta}\left(f + cs - \frac{\lambda\eta}{2}\right) + \delta p_r \theta\right] = 0$$

(24)

$$D(\phi, p) = (1 - p)\left[\left(\frac{d^2\phi}{d\eta^2} - \gamma\phi s_c\right) - \left(\frac{d^2\phi_I}{d\eta^2} - \gamma\phi_I s_c\right)\right] + p\left[\frac{d^2\phi}{d\eta^2} + \frac{d\phi}{d\eta}\left(f + cs - \frac{\lambda\eta}{2}\right)s_c - \gamma\phi s_c\right] = 0$$

(25)

With the following assumption

$$f = f_0 + pf_1 + p^2f_2 + \cdots,$$ (26)

$$s = s_0 + ps_1 + p^2s_2 + \cdots,$$ (27)

$$\theta = \theta_0 + p\theta_1 + p^2\theta_2 + \cdots$$ (28)

$$\phi = \phi_0 + p\phi_1 + p^2\phi_2 + \cdots$$ (29)

Using equation (26), (27), (28), (29) into equation (22), (23), (24) and (25) and on comparing the like powers of p, we get the zeoth order equation,

$$\left[\left(\frac{d^3f_0}{d\eta^3} - (\lambda - M)\frac{df_0}{d\eta}\right) - \left(\frac{d^3f_I}{d\eta^3} - (\lambda - M)\frac{df_I}{d\eta}\right)\right] = 0,$$ (30)

$$\left(\frac{d^3s_0}{d\eta^3} + (M - \lambda)\frac{ds_0}{d\eta}\right) - \left(\frac{d^3s_I}{d\eta^3} + (M - \lambda)\frac{ds_I}{d\eta}\right) = 0,$$ (31)

$$\left[\left(\frac{d^2\theta_0}{d\eta^2} + \delta p_r\theta_0\right) - \left(\frac{d^2\theta_I}{d\eta^2} + \delta p_r\theta_I\right)\right] = 0,$$ (32)

$$\left(\frac{d^2\phi_0}{d\eta^2} - \gamma\phi_0 s_c\right) - \left(\frac{d^2\phi_I}{d\eta^2} - \gamma\phi_I s_c\right) = 0.$$ (33)   with   the

corresponding boundary conditions are of zeroth order equations are:

$$\eta = 0: \theta_0 = 1, \phi_0 = 1, s_0' = 0, s_0 = 1, f_0 = -f_w, f_0' = 1, \;;$$

$$\eta = \infty: \theta_0 = 0, \phi_0 = 0, s_0' = 1, f_0' = 1,$$ (34)

And first order equations are:

$$\left(\frac{d^3f_1}{d\eta^3} + (M - \lambda)\frac{df_1}{d\eta}\right) - e^{-\eta}(f_w + M - \lambda - \lambda_1 - \lambda_2) - \eta e^{-\eta}\left(\frac{\lambda}{2} - c - 1\right) + ce^{-2\eta} = 0,$$ (35)

$$\left(\frac{d^3s_1}{d\eta^3} + (M - \lambda)\frac{ds_1}{d\eta}\right) - e^{-\eta}(2 + f_w - 2c - \lambda + M - \lambda_1 - \lambda_2) - \eta e^{-\eta}\left(\frac{\lambda}{2} - c - 1\right) + e^{-2\eta} = 0,$$

(36)

$$\frac{\partial^2\theta_I}{\partial\eta^2} + P_r\delta\theta_I + e^{-\eta} + P_re^{-\eta}\left(\eta + e^{-\eta} - f_w - 1 + c\eta + ce^{-\eta} - \frac{\lambda\eta}{2}\right) + P_r\delta e^{-\eta} = 0,$$ (37)

$$\frac{\partial^2\phi_1}{\partial\eta^2} - \gamma\phi_1 s_c + e^{-\eta} - e^{-\eta}\left(\eta + e^{-\eta} - f_w - 1 + c\eta + ce^{-\eta} - \frac{\lambda\eta}{2}\right)s_c - \gamma e^{-\eta}s_c = 0.$$ (38)

With the corresponding boundary conditions are of first order equations are:

$$\eta = 0: \theta_1 = 0, \phi_1 = 0, s_1' = 0, s_1 = 1, f_1 = 0, f_1' = 0, \;;$$

$$\eta = \infty: \theta_1 = 0, \phi_1 = 0, s_1^{'} = 1, f_1^{'} = 1, \tag{39}$$

Solving equations (30) to (33) and (35) to (38) under the corresponding boundary conditions, (34) and (39), letting $p \rightarrow 1$, values of eq.(26) to (29) are:

$$f = \eta + e^{-\eta} - f_w - 1 + \frac{c}{B_5[4-(B_5)^2]} - \frac{B_6}{B_5[1-(B_5)^2]} + \frac{B_6}{[1-(B_5)^2]} - \frac{c}{2[4-(B_5)^2]} - \frac{B_7}{(B_5)^2}\left[1 - \frac{1}{(B_5)^2} + \frac{2}{(B_5)^3}\right] +$$
$$\left[\frac{B_6}{B_5[1-(B_5)^2]} - \frac{c}{B_5[4-(B_5)^2]} + \frac{2B_7}{(B_5)^5}\right]e^{-B_5\eta} + \frac{ce^{-2\eta}}{2[4-(B_5)^2]} - \frac{B_6e^{-\eta}}{[1-(B_5)^2]} + \frac{B_7e^{-\eta}}{(B_5)^2}\left[\eta + 1 + \frac{\eta-1}{(B_5)^2}\right], \tag{40}$$

$$s = \eta + e^{-n} + \frac{B_1}{1-(B_3)^2}\left(1 - \frac{1}{B_3}\right) - \frac{1}{4-(B_3)^2}\left(\frac{1}{2} - \frac{1}{B_3}\right) - \frac{B_2}{B_3}\left(1 + \frac{2}{(B_3)^3} - \frac{1}{(B_3)^2}\right) + e^{-B_3\eta}\left(\frac{2B_2}{(B_3)^5} + \frac{B_1}{B_3(1-(B_3)^2)} -\right.$$
$$1B34-B32+e-2\eta24-B32-B1e-\eta1-B32+e-\eta B2B32\eta+1+\eta-1B32, \tag{41}$$

$$\theta = e^{-\eta} + \left[\frac{2A_3}{(A_1)^4} + \frac{A_2}{(A_1)^2+1} - \frac{A_4}{(A_1)^2+4}\right]e^{-A_1\eta} + \frac{e^{-2\eta}A_4}{(A_1)^2+4} - \frac{A_2e^{-\eta}}{(A_1)^2+1} - \frac{A_3e^{-\eta}}{(A_1)^2}\left[\eta - \frac{\eta}{(A_1)^2} + \frac{2}{(A_1)^2}\right] \tag{42}$$

$$\phi = e^{-\eta} + \left(\frac{A_6}{(A_5)^2-1} + \frac{A_8}{(A_5)^2-4} - \frac{2A_7}{(A_5)^4}\right)e^{-A_5\eta} - \frac{A_6e^{-\eta}}{(A_5)^2-1} - \frac{A_8e^{-2\eta}}{(A_5)^2-4} - \frac{A_7e^{-\eta}}{(A_5)^2}\left(\eta + \frac{\eta-2}{(A_5)^2}\right) \tag{43}$$

SKIN-FRICTION: Skin-friction coefficient at the sheet is given by
$$C_{fx}(Re_x)^{1/2} = 2\phi(\tau)f^{''}(0), \tag{44}$$

Table 1

λ	Skin friction coefficient	λ	Skin friction coefficient
0.1	1.784514	3.0	2.1021276
0.5	1.5623433	4.0	2.9498989
2.0	1.953478	5.0	3.623243

NUSSELT NUMBER: The rate of heat transfer in terms of the Nusselt number at the sheet is given by
$$(Re_x)^{-1/2}Nu_x = -\theta^{'}(0). \tag{45}$$

Table 2

λ	Nusselt number		
	M=0	M=1.0	M= 2.0
0.1	1.97856	0.123595	0.0010238
1.0	2.01856	3.262325	1.2512687
2.0	2.97856	3.223675	2.0236522

Conclusion: it is observed from table 1 and 2 that as λ and M increases, the numerical value of Skin friction coefficient $C_{fx}(Re_x)^{1/2}$ and Nusselt number $(Re_x)^{-1/2}Nu_x$ also increases. Figures 1,2,3 show the effects of the

ratio of the velocity gradients at the edge of the boundary layer c, suction/injection parameter f_w, and the magnetic field parameter M on stream function f, we observe that as c, f_w and M increases, value of **df** also increases. Figures 4,5,6 show the effects of the ratio of the velocity gradients at the edge of the boundary layer c, suction/injection parameter f_w, and the magnetic field parameter M on stream function S, we observe that as c, f_w and M increases, value of **ds** also increases. Figures7, 8, 9 and 10 show the temperature θ and concentration profiles ϕ increase with the decreasing of the ratio of velocity gradients at the edge of the boundary layer c, and the suction/injection parameter f_w. The above results obtained by HPM have good agreement with the results obtained by iterative tried diagonal inflict finite difference method

Fig 1
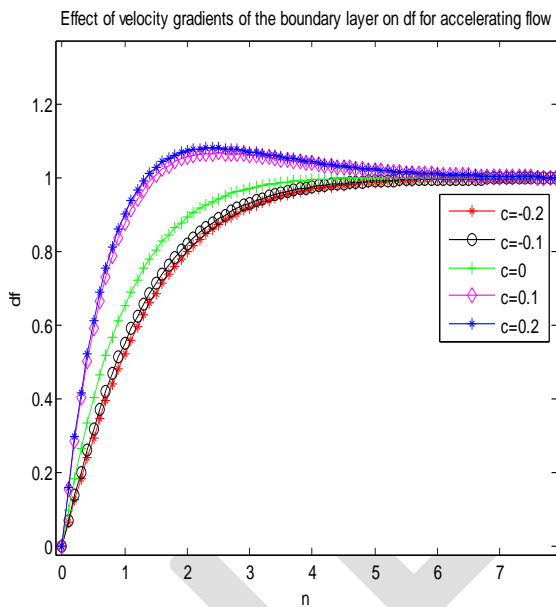
fig 2
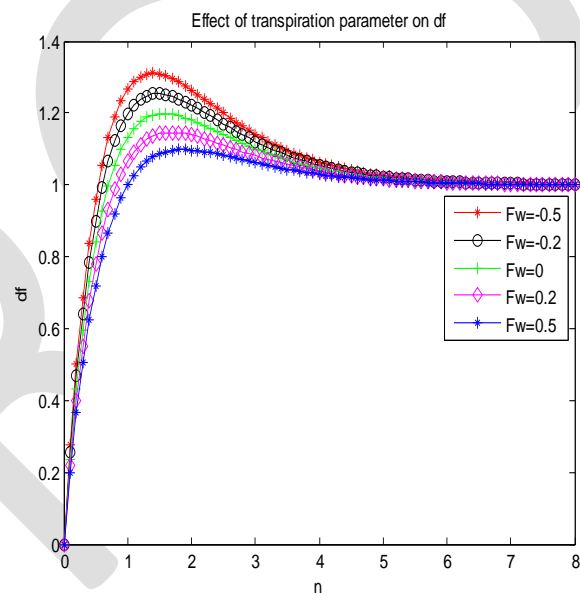
Fig 3
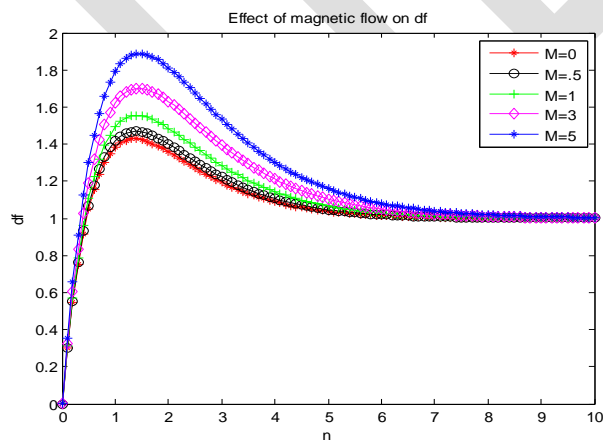
Fig 6
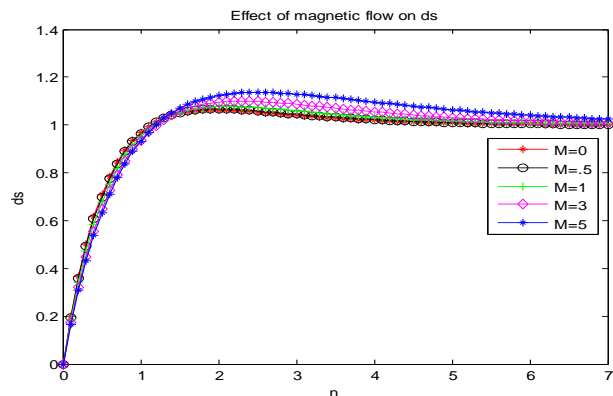
Fig 4

Effect of velocity gradients of the boundary layer on ds for the case of accelerating flow
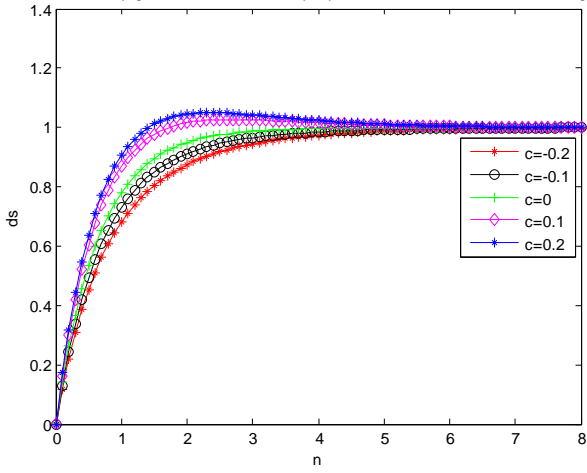
Fig 7

Effect velocity gradients of the boundary layer on ϕ for the case of accelerating flow
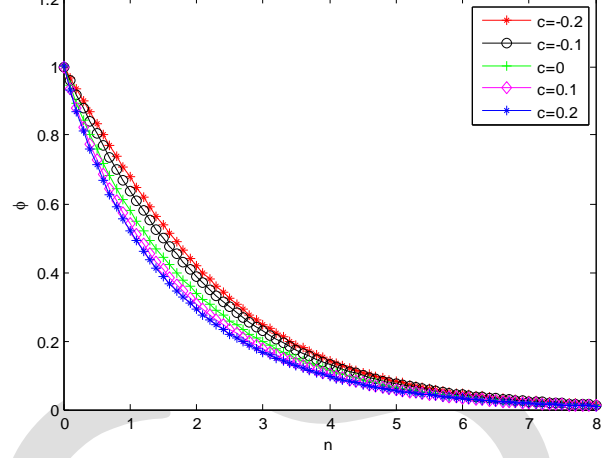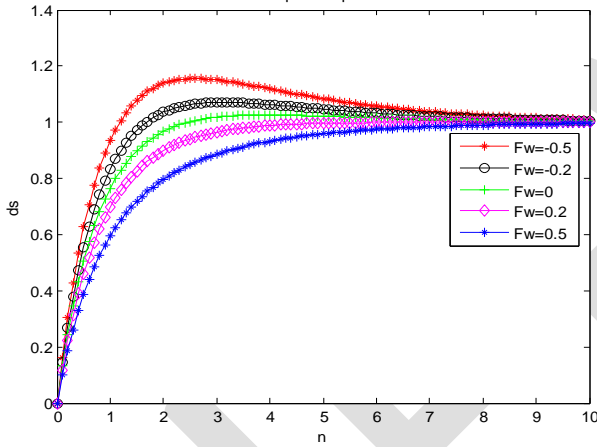
Fig 5

Effect of transpiration parameter on ds

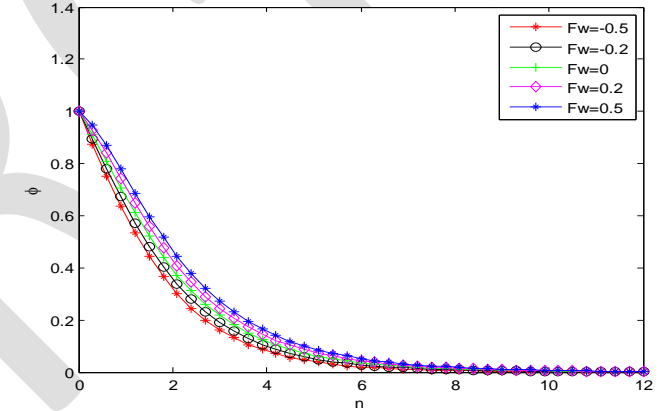Fig 8

Effect of transpiration parameter on ϕ

Fig 9

Effect velocity gradients of the boundary layer on θ for the case of accelerating flow
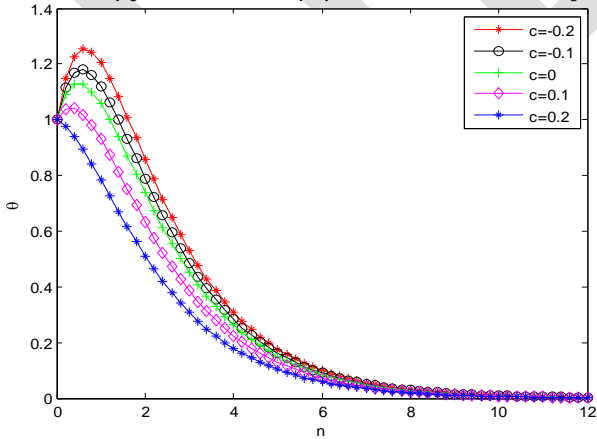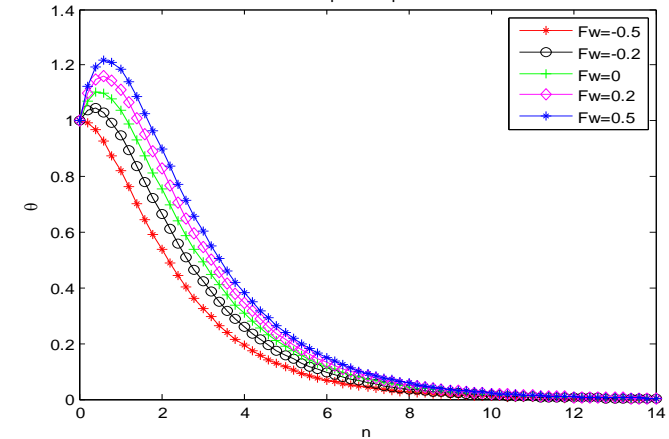
Fig 10

Effect of transpiration parameter on θ

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